As a general guide, base 2 is denoted by $(\quad)_2$. Base 3 is denoted by $(\quad)_3$ and so on. If we are in a system of base n, there is a carry over of 1 whenever we reach n. Likewise, if we reach $n + 1$, the number becomes 11. We look at some examples shown below.

$$(101)_2 + (111)_2 = (1100)_2$$
$$(223)_4 + (323)_4 = (1212)_4$$
$$(345)_6 + (555)_6 = (1344)_6$$

Compute the following.

(a) $(1001)_2 + (1100)_2$ **(b)** $(2341)_5 + (143)_5$

Advanced Example 1

Compute the following.

(a) $(1101)_2 \times (101)_2$ **(b)** $(10011)_2 \times (110)_2$

Advanced Example 2

Rewrite the numbers in base 10 and other bases in their expanded form.

(a) $(632)_{10} =$

(b) $(1864)_{10} =$

(c) $(7453)_{8} =$

(d) $(5241)_{6} =$

(e) $(1233)_{4} =$

(f) $(111001)_{2} =$

Advanced Example 3

Convert these numbers from base 10 to the base indicated below.

(a) $(8540)_{10} = (\qquad\qquad\qquad)_5$

(b) $(72)_{10} = (\qquad\qquad\qquad)_2$

Advanced Example 4

Convert these numbers from other bases to base 10.

(a) $(1234)_5 = (\qquad\qquad)_{10}$

(b) $(7746)_8 = (\qquad\qquad)_{10}$

Advanced Example 5

Compute the following.

(a) $(1011)_2 + (1111)_2$ **(b)** $(11011)_2 + (10011)_2$

(c) $(111111)_2 + (11011)_2$ **(d)** $(11111)_2 - (10011)_2$

Advanced Question 1

Convert these numbers from base 10 to the bases indicated below.

(a) $(1237)_{10} = (\qquad\qquad)_4$ **(b)** $(9653)_{10} = (\qquad\qquad)_8$

Advanced Question 2

Rewrite these numbers in base 10 or other bases in expanded form.

(a) $(894)_{10}$

(b) $(17653)_{10}$

(c) $(4321)_5$

(d) $(6544)_7$

(e) $(8888)_9$

(f) $(7546)_8$

Advanced Question 3

Convert these numbers in other bases to base 10.

(a) $(10111)_2 = (\qquad\qquad)_{10}$

(b) $(100100)_2 = (\qquad\qquad)_{10}$

(c) $(123123)_4 = (\qquad\qquad)_{10}$

(d) $(443322)_5 = (\qquad\qquad)_{10}$

Advanced Question 4

Complete the table below.

base 10	2	( )	6	( )	10
base 3	( )	11	( )	22	( )

Advanced Question 5

Convert these number in other bases to base 10.

(a) $(1202)_3 = (\qquad\qquad)_{10}$

(b) $(4321)_5 = (\qquad\qquad)_{10}$

Advanced Question 6

Complete the base 6 times table.

	1	2	3	4	5
1	1	2	3	4	5
2		4	10	12	14
3			13	20	23
4				24	32
5					41

Advanced Question 7

Find the number base in which

$$\begin{array}{r} \text{A B C D} \\ +\ \text{B D C A} \\ \hline \text{D B 0 0 0} \end{array}$$

is computed.

Advanced Question 8

Complete the base 8 times table

	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2		4	6	10	12	14	16
3			11	14	17	22	25
4				20	24	30	34
5					31	36	43
6						44	52
7							61

Advanced Question 9

Given $(122)_4 = (abc)_5$, find the number represented by abc.

Advanced Question 10

Given $(724)_8 = (abc)_9$, find the value represented by abc.

Advanced Question 11

A number in base 10 can be expressed as $(abc)_3$.
It can also be expressed as $(cba)_4$.
Find the number.

Advanced Question 12

Solution for Advanced Example 1

(a) $(1001)_2 + (1100)_2$

$= \mathbf{(10101)_2}$

$$\begin{array}{r} 1\,0\,0\,1 \\ +\ 1\,1\,0\,0 \\ \hline 1\,0\,1\,0\,1 \\ \hline \end{array}$$

Notice in base 2, (1 + 1) is expressed as 10.

(b) $(2341)_5 + (143)_5$

$= \mathbf{(3034)_5}$

$$\begin{array}{r} 2\,3\,4\,1 \\ +\ \ 1\,4\,3 \\ \hline 3\,0\,3\,4 \\ \hline \end{array}$$

Notice in base 5, (4 + 4) is expressed as 13.

Solution for Advanced Example 2

(a) $(1101)_2 \times (101)_2$

$= \mathbf{(1000001)_2}$

$$\begin{array}{r} 1\,1\,0\,1 \\ \times\ \ 1\,0\,1 \\ \hline 1\,1\,0\,1 \\ +\ 1\,1\,0\,1\ \ \ \ \\ \hline 1\,0\,0\,0\,0\,0\,1 \\ \hline \end{array}$$

(b) $(10011)_2 \times (110)_2$

$= \mathbf{(1110010)_2}$

$$\begin{array}{r} 1\,0\,0\,1\,1 \\ \times\ \ 1\,1\,0 \\ \hline 1\,0\,0\,1\,1\,0 \\ +\ 1\,0\,0\,1\,1\ \ \ \ \\ \hline 1\,1\,1\,0\,0\,1\,0 \\ \hline \end{array}$$

Note that (1 + 1) is expressed as 10 in both (a) and (b).

Solution for Advanced Example 3

(a) $(632)_{10} = 6 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$ $(10^0 = 1)$

(b) $(1864)_{10} = 1 \times 10^3 + 8 \times 10^2 + 6 \times 10^1 + 4 \times 10^0$ $(10^0 = 1)$

(c) $(7453)_8 = 7 \times 8^3 + 4 \times 8^2 + 5 \times 8^1 + 3 \times 8^0$ $(8^0 = 1)$

(d) $(5241)_6 = 5 \times 6^3 + 2 \times 6^2 + 4 \times 6^1 + 1 \times 6^0$ $(6^0 = 1)$

(e) $(1233)_4 = 1 \times 4^3 + 2 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$ $(4^0 = 1)$

(f) $(111001)_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^0$ $(2^0 = 1)$

Solution for Advanced Example 4

(a)

$$\begin{array}{rll} 5 & \underline{|\,8\,5\,4\,0} & \\ 5 & \underline{|\,1\,7\,0\,8} & \text{— R0} \\ 5 & \underline{|\,3\,4\,1} & \text{— R3} \\ 5 & \underline{|\,6\,8} & \text{— R1} \uparrow \\ 5 & \underline{|\,1\,3} & \text{— R3} \\ & 2 & \rightarrow \text{R3} \end{array}$$

$(8540)_{10} = \mathbf{(233130)_5}$

(b)

$$\begin{array}{rll} 2 & \underline{|\,7\,2} & \\ 2 & \underline{|\,3\,6} & \text{— R0} \\ 2 & \underline{|\,1\,8} & \text{— R0} \\ 2 & \underline{|\,9} & \text{— R0} \uparrow \\ 2 & \underline{|\,4} & \text{— R1} \\ 2 & \underline{|\,2} & \text{— R0} \\ & 1 & \rightarrow \text{R0} \end{array}$$

$(72)_{10} = \mathbf{(1001000)_2}$

Solution for Advanced Example 5

(a) $(1234)_5 = 1 \times 5^3 + 2 \times 5^2 + 3 \times 5^1 + 4 \times 5^0$

$= 125 + 50 + 15 + 4$

$= \mathbf{(194)_{10}}$

(b) $(7746)_8 = 7 \times 8^3 + 7 \times 8^2 + 4 \times 8^1 + 6 \times 8^0$

$= 3584 + 448 + 32 + 6$

$= \mathbf{(4070)_{10}}$

Solution for Advanced Question 1

(a)

$$\begin{array}{r} 1\,0\,1\,1 \\ +\ 1\,1\,1\,1 \\ \hline 1\,1\,0\,1\,0 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 1\,1\,0\,1\,1 \\ +\ 1\,0\,0\,1\,1 \\ \hline 1\,0\,1\,1\,1\,0 \\ \hline \end{array}$$

(c)

$$\begin{array}{r} 1\,1\,1\,1\,1\,1 \\ +\ \ 1\,1\,0\,1\,1 \\ \hline 1\,0\,1\,1\,0\,1\,0 \\ \hline \end{array}$$

(d)

$$\begin{array}{r} 1\,1\,1\,1\,1 \\ -\ 1\,0\,0\,1\,1 \\ \hline 1\,1\,0\,0 \\ \hline \end{array}$$

Solution for Advanced Question 2

(a)

$$\begin{array}{rll} 4 & \underline{|\,1\,2\,3\,7} & \\ 4 & \underline{|\,3\,0\,9} & \text{— R1} \\ 4 & \underline{|\,7\,7} & \text{— R1} \\ 4 & \underline{|\,1\,9} & \text{— R1} \uparrow \\ 4 & \underline{|\,4} & \text{— R3} \\ & 1 & \longrightarrow \text{R0} \end{array}$$

$(1237)_{10} = \mathbf{(103111)_4}$

(b)

$$\begin{array}{rll} 8 & \underline{|\,9\,6\,5\,3} & \\ 8 & \underline{|\,1\,2\,0\,6} & \text{— R5} \\ 8 & \underline{|\,1\,5\,0} & \text{— R6} \\ 8 & \underline{|\,1\,8} & \text{— R6} \uparrow \\ 8 & \underline{|\,2} & \text{— R2} \\ & 0 & \longrightarrow \text{R2} \end{array}$$

$(9653)_{10} = \mathbf{(22665)_8}$

Solution for Advanced Question 3

(a) $(894)_{10} = 8 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$

(b) $(17653)_{10} = 1 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 3 \times 10^0$

(c) $(4321)_5 = 4 \times 5^3 + 3 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$

(d) $(6544)_7 = 6 \times 7^3 + 5 \times 7^2 + 4 \times 7^1 + 4 \times 7^0$

(e) $(8888)_9 = 8 \times 9^3 + 8 \times 9^2 + 8 \times 9^1 + 8 \times 9^0$

(f) $(7546)_8 = 7 \times 8^3 + 5 \times 8^2 + 4 \times 8^1 + 6 \times 8^0$

Solution for Advanced Question 4

(a) $(10111)_2 = 1\times2^4 + 1\times2^2 + 1\times2^1 + 1\times2^0$
$= 16 + 4 + 2 + 1 = \mathbf{(23)_{10}}$

(b) $(100100)_2 = 1\times2^5 + 1\times2^2$
$= 32 + 4 = \mathbf{(36)_{10}}$

(c) $(123123)_4$
$= 1\times4^5 + 2\times4^4 + 3\times4^3 + 1\times4^2 + 2\times4^1 + 3\times4^0$
$= 1024 + 512 + 192 + 16 + 8 + 3$
$= \mathbf{(1755)_{10}}$

(d) $(443322)_5$
$= 4\times5^5 + 4\times5^4 + 3\times5^3 + 3\times5^2 + 2\times5^1 + 2\times5^0$
$= 12\,500 + 2500 + 375 + 75 + 10 + 2$
$= \mathbf{(15\,462)_{10}}$

Solution for Advanced Question 5

Base 10	2	4	6	8	10
Base 3	2	11	20	22	101

Solution for Advanced Question 6

(a) $(1202)_3 = 1\times3^3 + 2\times3^2 + 2\times3^0 \quad (3^0 = 1)$
$= 27 + 18 + 2$
$= \mathbf{(47)_{10}}$

(b) $(4321)_5$
$= 4\times5^3 + 3\times5^2 + 2\times5^1 + 1\times5^0 \quad (5^0 = 1)$
$= 500 + 75 + 10 + 1$
$= \mathbf{(586)_{10}}$

Solution for Advanced Question 7

	1	2	3	4	5
1	1	2	3	4	5
2	2	4	10	12	14
3	3	10	13	20	23
4	4	12	20	24	32
5	5	14	23	32	41

Solution for Advanced Question 8

$$\begin{array}{r} 4\,3\,2\,1 \\ +\ 3\,1\,2\,4 \\ \hline 1\,3\,0\,0\,0 \\ \hline \end{array}$$

This computation is based on **base 5.**

Solution for Advanced Question 9

	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	10	12	14	16
3	3	6	11	14	17	22	25
4	4	10	14	20	24	30	34
5	5	12	17	24	31	36	43
6	6	14	22	30	36	44	52
7	7	16	25	24	43	52	61

Solution for Advanced Question 10

$(122)_4 = 1\times4^2 + 2\times4^1 + 2\times4^0$
$= 16 + 8 + 2$
$= (26)_{10}$

$$\begin{array}{r|l} 5 & 26 \\ \hline 5 & 5 \ -\ R1 \\ \hline & 1 \ \rightarrow R0 \end{array}$$

abc = 101

The number represented by abc is **101.**

Solution for Advanced Question 11

$(724)_8 = 7\times8^2 + 2\times8^1 + 4\times8^0$
$= 448 + 16 + 4$
$= (468)_{10}$

$$\begin{array}{r|l} 9 & 468 \\ \hline 9 & 52 \ -\ R0 \\ \hline 9 & 5 \ -\ R7 \\ \hline & 0 \ -\ R5 \end{array}$$

abc = 570

The value represented by abc is **570.**

Solution for Advanced Question 12

$(abc)_3 = a\times3^2 + b\times3^1 + c\times3^0$
$= 9a + 3b + c$

$(cba)_4 = c\times4^2 + b\times4^1 + a\times4^0$
$= 16c + 4b + a$

Equating,

$9a + 3b + c = 16c + 4b + a$
$8a - b - 15c = 0$
$8a = b + 15c$

We are bound by base 3 and base 4.

$a = 2, b = 1, c = 1$

The number is **211.**