The concept of average is used extensively in our daily life. For example, the average class score for the mathematics test, science final examination, pupils' average height and etc.

The formulae to find average and total are shown below.

average = total ÷ number of units or events

total = average × number of units or events

An interesting fact on average is shown below.

$1 + 2 + 3 + \cdots + 49 + 50$
= average of the first and last terms × number of terms

Daniel's average marks for the first three tests in a year was 73. He worked very hard for the last test in that year to bring the average mark to 75. How many marks should he score for the last test in order to achieve this?

Intermediate Example 1

1

The sum of eight consecutive whole numbers is 188.
List all the eight numbers.

It took Samuel 18 min to walk to the library at a speed
of 40 m/min. He returned from the library at a walking
speed of 60 m/min. What was his average speed for
the whole trip?

The average of A and B is 20. The average of B and C
is 15. The average of C and D is 18. Find the average
of A and D.

2

The average mass of three people in a lift was 60 kg. After another person had come into the lift, the average mass became 57 kg. What was the mass of the last person?

The average of three number is 120. What number must be added so that the average will become 110?

The table below shows the scores of six students in the second Continual Assessment.

Danny	Alison	Peter	John	Damien	Melissa
77	82	78	95	83	75

What is the average score in the Continual Assessment without considering the highest and the lowest scores?

A box of thumbnails weighed 1590 g. The box of thumbnails weighed 1470 g after 40 of them were taken out from the box. How many thumbnails were in the box at first?

The average of five numbers is 20. The average of the five numbers is 18 when one of the numbers is changed to 4. What is the original value of the changed number?

The average of A, B and C is 70. The value of A is 2 more than B. The value of B is 11 more than C. Find the values of A, B and C.

Julie read 83 pages of a storybook on the first day, 66 pages on the second day, 74 pages on the third day and 73 pages on the fourth day. The number of pages she read on the fifth day was 12 pages more than the average number of pages she had read during the first four days. How many pages did she read on the fifth day?

The average of A and B is 50.
The average of B and C is 43.
The average of A and C is 45.
Find the average of A, B and C.

The average mass of Ken, David and Eugene is 42 kg. David is 6 kg heavier than the average mass of Ken and Eugene. Ken is 6 kg heavier than Eugene. Find David's mass.

Matthew needs to get a perfect score of 100 for his last English test in this year so as to improve on his average score from 84 to 86. How many English tests were there altogether in this year?

A car travelled from Town A to Town B at a speed of 30 km/h. The driver returned from Town B in the same car at a speed of 60 km/h. What was the average driving speed for the two trips?

The average of A and B is 8. The average of B and C is 3.6.
The average of C and D is 5.8. Find the average of A and D.

The average mass of a group of children is 36 kg. If $\frac{3}{7}$ of the
number of children are girls and their average mass is 32 kg.
Find the average mass of the boys.
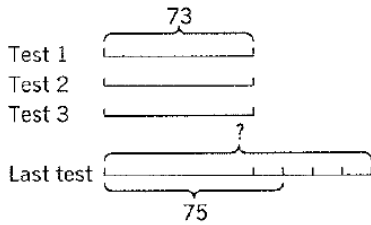
Solution for Intermediate Example 1

Method 1: Solve by Reasoning

Total marks for the first three tests = $3 \times 73 = 219$
Total marks for the whole year = $4 \times 75 = 300$

$$300 - 219 = 81$$

Method 2: Solve by Drawings
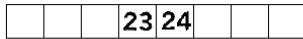

$$75 - 73 = 2$$
$$2 \times 3 = 6$$
$$6 + 75 = 81$$

He should score **81** marks for the last test in order to achieve it.

Solution for Intermediate Example 2

$$188 \div 8 = 23.5$$

The middle pair of numbers are 23 and 24.

			23	24			

The eight numbers are **20, 21, 22, 23, 24, 25, 26** and **27**.

Solution for Beginner Example 3

$$40 \times 18 = 720 \text{ m}$$

The library was 720 m away.

$$720 \div 60 = 12 \text{ min}$$

He took 12 min to return from the library.

$$18 + 12 = 30 \text{ min}$$

He took 30 min for the whole trip.

$$720 \times 2 = 1440 \text{ m}$$

The total distance for the whole trip was 1440 m.

$$1440 \div 30 = 48$$

His average speed for the whole trip was **48 m/min**.

Solution for Beginner Example 4

$$\frac{A+B}{2} = 20 \quad \cdots\cdots\cdots\cdots (1)$$
$$\frac{B+C}{2} = 15 \quad \cdots\cdots\cdots\cdots (2)$$
$$\frac{C+D}{2} = 18 \quad \cdots\cdots\cdots\cdots (3)$$

Simplifying all equations,

$$A + B = 40 \cdots\cdots\cdots\cdots (1)$$
$$B + C = 30 \cdots\cdots\cdots\cdots (2)$$
$$C + D = 36 \cdots\cdots\cdots\cdots (3)$$

$(1) - (2)$
$$A + B - (B + C) = 40 - 30$$
$$A + B - B - C = 10$$
$$A - C = 10 \cdots\cdots\cdots\cdots (4)$$

$(4) + (3)$
$$A - C + C + D = 10 + 36$$
$$A + D = 46$$
$$\frac{A+D}{2} = 23$$

The average of A and D is **23**.

Solution for Intermediate Question 1

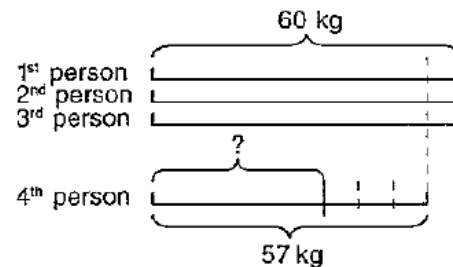*Method 1: Solve By Reasoning*

Total mass for 3 people
$= 3 \times 60 = 180$ kg
Total mass for 4 people
$= 4 \times 57 = 228$ kg

$$228 - 180 = 48 \text{ kg}$$

Method 2: Solve by Drawings

$$60 - 57 = 3$$
$$3 \times 3 = 9$$
$$57 - 9 = 48$$

The mass of the last person was **48 kg**.

Solution for Intermediate Question 2

$$3 \times 120 = 360$$
$$4 \times 110 = 440$$
$$440 - 360 = 80$$

The number, **80**, must be added so that the average will become 110.

Solution for Intermediate Question 3

Highest = 95   Lowest = 75

$$\frac{77 + 82 + 78 + 83}{4} = 80$$

The average score in the Continual Assessment without considering the highest and the lowest scores is **80**.

Solution for Intermediate Question 4

$$1590 - 1470 = 120$$
$$120 \div 40 = 3$$
Each thumbnail weighs 3 g.
$$1590 \div 3 = 530$$
There were **530** thumbnails in the box at first

Solution for Intermediate Question 5

$$5 \times 20 = 100$$
$$5 \times 18 = 90$$
$$100 - 90 = 10$$
$$10 + 4 = 14$$

The original value of the changed number is **14**.

Solution for Intermediate Question 7

$A = 2 + B$ ·························(1)
$B = 11 + C$ ·····················(2)
From (2), $C = B - 11$ ······(3)

$$A + B + C = 3 \times 70$$
$$2 + B + B + B - 11 = 210$$
$$3B = 210 + 11 - 2$$
$$3B = 219$$
$$B = 219 \div 3 = 73$$
$$A = 73 + 2 = 75$$
$$C = 73 - 11 = 62$$

The values of A, B and C are **75**, **73** and **62** respectively.

Solution for Intermediate Question 8

$$\frac{83 + 66 + 74 + 73}{4} = 74$$

$$74 + 12 = 86$$

She read **86** pages on the fifth day.

Solution for Intermediate Question 9

$$\frac{A + B}{2} = 50 \qquad \frac{B + C}{2} = 43$$

$$\frac{A + C}{2} = 45$$

$$A + B = 100$$
$$B + C = 86$$
$$A + C = 90$$
$$2A + 2B + 2C = 100 + 86 + 90 = 276$$
$$A + B + C = 276 \div 2 = 138$$
$$\frac{A + B + C}{3} = 138 \div 3 = 46$$

The average of A, B and C is **46**.

Solution for Intermediate Question 11

Let Eugene's mass be m.
Eugene = m
Ken   = m + 6

David $= \dfrac{m + m + 6}{2} + 6 = \dfrac{2m + 6}{2} + 6$
$$= m + 3 + 6$$
$$= m + 9$$

$$m + m + 6 + m + 9 = 126$$
$$3m = 126 - 6 - 9 = 111$$
$$m = 111 \div 3 = 37$$

$$37 + 9 = 46$$

David's mass is **46 kg**.

Solution for Intermediate Question 12

$$100 - 86 = 14 \text{ marks}$$

14 marks are needed for the average score to move from 84 to 86.

$$86 - 84 = 2$$
$$14 \div 2 = 7$$
$$7 + 1 = 8$$

There were **8** English tests altogether in this year.

Solution for Intermediate Question 13

common multiple of 30 and 60 = 180 km

Time taken to travel from Town A to Town B = 180 ÷ 30 = 6 h

Time taken to return from Town B = 180 ÷ 60 = 3 h

Total time for the two trips = 6 + 3 = 9

Total distance = 2 × 180 = 360 km

$$360 ÷ 9 = 40 \text{ km/h}$$

The average driving speed for the two trips was **40 km/h**.

Solution for Intermediate Question 14

$$\frac{A+B}{2} = 8 \qquad\qquad \frac{B+C}{2} = 3.6$$

$$\frac{C+D}{2} = 5.8$$

$$
\begin{aligned}
A + B &= 16 & &\cdots\cdots\cdots (1)\\
B + C &= 7.2 & &\cdots\cdots\cdots (2)\\
C + D &= 11.6 & &\cdots\cdots\cdots (3)
\end{aligned}
$$

(1) − (2)

$$A + B - (B + C) = 16 - 7.2$$
$$A - C = 8.8 \qquad \cdots\cdots\cdots (4)$$

(4) + (3)

$$\cdot\ A - C + C + D = 8.8 + 11.6$$
$$A + D = 20.4$$

$$\frac{A+D}{2} = 20.4 ÷ 2 = 10.2$$

The average of A and D is **10.2**.

Solution for Intermediate Question 15

Total mass = 7 × 36 = 252 kg
Total mass of girls = 3 × 32 = 96 kg
Total mass of boys = 252 − 96 = 156 kg

$$156 ÷ 4 = 39 \text{ kg}$$

The average mass of the boys is **39 kg**.