What's the Difference?

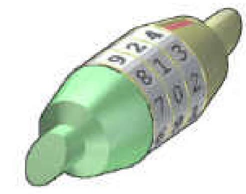
In English we use the word "combination" loosely, without thinking if the **order** of things is important. In other words:

"My fruit salad is a combination of apples, grapes and bananas" We don't care what order the fruits are in, they could also be "bananas, grapes and apples" or "grapes, apples and bananas", its the same fruit salad.

"The combination to the safe is 472". Now we **do** care about the order. "724" won't work, nor will "247". It has to be exactly **4-7-2**.

So, in Mathematics we use more *accurate* language:

- If the order doesn't matter, it is a **Combination**.
- If the order **does** matter it is a **Permutation**.



So, we should really call this a "Permutation Lock"!

In other words: A Permutation is an **ordered** Combination.

A few examples

Here's a few examples of combinations (order doesn't matter) from permutations (order matters).

- Combination: Picking a team of 3 people from a group of 10. C(10,3) = 10!/(7! * 3!) = 10 * 9 * 8 / (3 * 2 * 1) = 120.

  Permutation: Picking a President, VP and Waterboy from a group of 10. P(10,3) = 10!/7! = 10 * 9 * 8 = 720.

- Combination: Choosing 3 desserts from a menu of 10. C(10,3) = 120.

  Permutation: Listing your 3 favorite desserts, in order, from a menu of 10. P(10,3) = 720.

Don't memorize the formulas, understand why they work. **Combinations sound simpler than permutations, and they are. You have fewer combinations than permutations.**

Permutations

There are basically two types of permutation:

- **Repetition is Allowed**: such as the lock above. It could be "333".
- **No Repetition**: for example the first three people in a running race. You can't be first *and* second.

1. Permutations with Repetition

These are the easiest to calculate.

When we have ***n*** things to choose from ... we have ***n*** choices each time!

When choosing ***r*** of them, the permutations are:

n × n × ... *(r times)*

(In other words, there are **n** possibilities for the first choice, THEN
there are **n** possibilites for the second choice, and so on,
multplying each time.)

Which is easier to write down using an exponent of **r**:

$$n \times n \times \ldots \text{ (r times)} = n^r$$

Example: in the lock above, there are 10 numbers to choose from (0,1,...9) and we choose 3 of them:

$$10 \times 10 \times \ldots \text{ (3 times)} = 10^3 = 1{,}000 \text{ permutations}$$

Permutations

There are basically two types of permutation:

- **Repetition is Allowed**: such as the lock above. It could be "333".
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2. Permutations without Repetition

For example, imagine putting the letters *a, b, c, d* into a hat, and then drawing two of them in succession. We can draw the first in 4 different ways: either *a* or *b* or *c* or *d*. After that has happened, there are 3 ways to choose the second. That is, to *each* of those 4 ways there correspond 3. Therefore, there are $4 \cdot 3$ or 12 possible ways to choose two letters from four.

ab	*ba*	*ca*	*da*
ac	*bc*	*cb*	*db*
ad	*bd*	*cd*	*dc*

ab means that *a* was chosen first and *b* second; *ba* means that *b* was chosen first and *a* second; and so on.

Let us now consider the total number of permutations of all four letters. There are 4 ways to choose the first. 3 ways remain to choose the second, 2 ways to choose the third, and 1 way to choose the last. Therefore the number of permutations of 4 different things is

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Thus the number of permutations of 4 different things taken 4 at a time is 4!.

Permutations of less than all

We have seen that the number of ways of choosing 2 letters from 4 is $4\cdot 3 = 12$. We call this

"The number of permutations of 4 different things taken 2 at a time."

We will symbolize this as ${}^4P_2$:

$${}^4P_2 = 4\cdot 3$$

The lower index 2 indicates the number of factors. The upper index 4 indicates the first factor.

For example, ${}^8P_3$ means "the number of permutations of 8 different things taken 3 at a time." And

$$\begin{aligned} {}^8P_3 &= 8\cdot 7\cdot 6 \\ &= 56\cdot 6 \\ &= 50\cdot 6 + 6\cdot 6 \\ &= 336 \end{aligned}$$

For, there are 8 ways to choose the first, 7 ways to choose the second, and 6 ways to choose the third.

Problem : a) How many different arrangements (permutations) are there of the digits 01234?

$5! = 120$

b) How many 5-digit numbers can you make of those digits, in which the first digit is not 0, and no digit is repeated?

1st digit = cannot be 0, there will be 4 ways
For 2nd to 5th digit, 0 can be used again.
2nd digit = 4 ways
3rd digit = 3 ways
4th digit = 2 ways
5th digit = 1 way
The total number of ways = $4 \times 4! = 96$

c) How many 5-digit *odd* numbers can you make with 0, 1, 2, 3, 4, and

1st digit = cannot be 0
To be Odd number, the last digit must be either 1 or 3.
Scenario 1: _ _ _ _ 1
1st digit = 3 ways (minus 0 and 1)
For 2nd to 4th digit, 0 can be used again
2nd digit = 3 ways
3rd digit = 2 ways
4th digit = 1 way
5th digit = 1 way
Number of ways = $3 \times 3! = 18$
Scenario 1: _ _ _ _ 3
1st digit = 3 ways (minus 0 and 3)
For 2nd to 4th digit, 0 can be used again
2nd digit = 3 ways
3rd digit = 2 ways
4th digit = 1 way
5th digit = 1 way
Number of ways = $3 \times 3! = 18$
Total number of odd number = $18 + 18 = 36$

Permutations with Some Identical Elements

1. How many **different** 5-letter words can be formed from the word APPLE ?

$$\frac{{}_5P_5}{2!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = \frac{120}{2} = \textbf{60 words}$$

You divide by 2! because the letter P repeats twice.

2. How many **different** six-digit numerals can be written using all of the following six digits: 4,4,5,5,5,7

$$\frac{{}_6P_6}{2!3!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1} = \frac{720}{12} = \textbf{60}$$

Two fours repeat and three fives repeat, so divide by 2! and 3!